Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach

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Outline

Outline

Introduction

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  - Fixed cost in production
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  - Total factor productivity shock
  - Risk premium shock
  - Investment-specific technology shock
  - Wage mark-up shock
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  - Exogenous spending shock
  - Monetary policy shock

Outline

Framework

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- Final goods producers
  - Produce final goods
- Labor unions
  - Produce intermediate labor, set wage
- Labor packers
  - Produce final labor
- Government
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Final Goods Producers

- Profit maximization

$$\max_{Y_t, Y_{i,t}} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

$$s.t. \int_0^1 G\left(\frac{Y_{i,t}}{Y_t}; \varepsilon_t^p\right) di = 1 \tag{1}$$

- Price mark-up shock

$$\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p; \quad \eta_t^p \sim N(0, \sigma_p^2)$$

Intermediate Goods Producers

- Cost minimization

$$\min_{K^s_{i,t}, L_{i,t}} W_t L_{i,t} + R^k_t K^s_{i,t}$$

$$s.t.\ Y_{i,t} = \mathrm{e}^{\varepsilon^a_t} (K^s_{i,t})^{\alpha} (\gamma^t L_{i,t})^{1-\alpha} - \gamma^t \Phi \tag{2}$$

- Total factor productivity shock

$$\varepsilon^a_t = \rho_a \varepsilon^a_{t-1} + \eta^a_t; \quad \eta^a_t \sim N(0, \sigma^2_a)$$

- Price setting

$$\max_{\tilde{P}_{i,t}} E_t \sum_{s=0}^{\infty} \xi^s_p \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} [\tilde{P}_{i,t} X^p_{t,s} - MC_{t+s}] Y_{i,t+s}$$

$$s.t.\ Y_{i,t+s} = Y_{t+s} G'^{-1} \left(\frac{\tilde{P}_{i,t} X^p_{t,s}}{P_{t+s}} \tau^p_{t+s} \right)$$

where

$$X^p_{t,s} = \begin{cases} 1 & \text{for } s = 0 \\ \prod_{l=1}^{s} (\pi^{\iota_p}_{t+l-1} \pi^{1-\iota_p}) & \text{for } s = 1, \ldots, \infty \end{cases}$$

and

$$\tau^p_{t+s} = \int_0^1 G' \left(\frac{Y_{i,t+s}}{Y_{t+s}}; \varepsilon^p_{t+s} \right) \frac{Y_{i,t+s}}{Y_{t+s}} di$$

Household

- Utility maximization

$$\max_{C_t, I_t, B_t, Z_t, \bar{L}_t} E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{1-\sigma_c} (C_{t+s} - \lambda \bar{C}_{t+s-1})^{1-\sigma_c} \right] \mathrm{e}^{\frac{\sigma_c - 1}{1+\sigma_l} \bar{L}_{t+s}^{1+\sigma_l}}$$

$$s.t. \; C_{t+s} + I_{t+s} + \frac{B_{t+s}}{\mathrm{e}^{\varepsilon^b_{t+s}} R_{t+s} P_{t+s}} + T_{t+s} \leq \frac{B_{t+s-1}}{P_{t+s}} + \frac{W^h_{t+s} \bar{L}_{t+s}}{P_{t+s}} + \frac{R^k_{t+s} Z_{t+s} K_{t+s-1}}{P_{t+s}} \quad (3)$$

$$- a(Z_{t+s}) K_{t+s-1} + \frac{Div_{t+s}}{P_{t+s}}$$

$$K_{t+s} = (1-\delta) K_{t+s-1} + \mathrm{e}^{\varepsilon^i_{t+s}} \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}} \right) \right] I_{t+s} \quad (4)$$

where

$$K^s_{t+s} = Z_{t+s} K_{t+s-1} \quad (5)$$

and

$$\bar{L}_{t+s} = \int_0^1 L_{l,t+s} dl$$

- Risk premium shock

$$\varepsilon^b_t = \rho_b \varepsilon^b_{t-1} + \eta^b_t; \quad \eta^b_t \sim N(0, \sigma^2_b)$$

- Investment-specific technology shock

$$\varepsilon^i_t = \rho_i \varepsilon^i_{t-1} + \eta^i_t; \quad \eta^i_t \sim N(0, \sigma^2_i)$$

Labor Sector

- Labor packers (final labor provider)

$$\max_{L_t, L_{l,t}} W_t L_t - \int_0^1 W_{l,t} L_{l,t} dl$$

$$s.t. \int_0^1 H\left(\frac{L_{l,t}}{L_t}; \varepsilon_t^w\right) dl = 1$$

- Wage mark-up shock

$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w; \quad \eta_t^w \sim N(0, \sigma_w^2)$$

- Labor unions (intermediate labor provider)

$$\max_{\tilde{W}_{l,t}} E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} [\tilde{W}_{l,t} X_{t,s}^w - W_{t+s}^h] L_{l,t+s}$$

$$s.t. \; L_{l,t+s} = L_{t+s} H'^{-1}\left(\frac{\tilde{W}_{l,t} X_{t,s}^w}{W_{t+s}} \int_0^1 H'\left(\frac{L_{l,t+s}}{L_{t+s}}; \varepsilon_{t+s}^w\right) \frac{L_{l,t+s}}{L_{t+s}} dl\right)$$

where

$$X_{t,s}^w = \begin{cases} 1 & \text{for } s = 0 \\ \prod_{l=1}^{s} (\gamma \pi_{t+l-1}^{\iota_w} \pi^{1-\iota_w}) & \text{for } s = 1, \ldots, \infty \end{cases}$$

Government

- Monetary policy

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho} \left[\left(\frac{\pi_t}{\pi}\right)^{r_\pi} \left(\frac{Y_t}{Y_t^p}\right)^{r_y}\right]^{1-\rho} \left(\frac{Y_t/Y_{t-1}}{Y_t^p/Y_{t-1}^p}\right)^{r_{\Delta y}} \mathrm{e}^{\varepsilon_t^r} \tag{6}$$

where Y_t^p is potential output (output under flexible prices and wages in the absence of the two "mark-up" shocks).

- Monetary policy shock

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r; \quad \eta_t^r \sim N(0, \sigma_r^2)$$

- Government budget

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t}$$

where

$$G_t = \mathrm{e}^{\varepsilon_t^g} g_y y \gamma^t$$

- Exogenous spending shock

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a; \quad \eta_t^g \sim N(0, \sigma_g^2)$$

Final Goods Producers

- F.O.C.

$$Y_{i,t} = Y_t G'^{-1} \left[\frac{P_{i,t}}{P_t} \int_0^1 G' \left(\frac{Y_{j,t}}{Y_t} ; \varepsilon_t^p \right) \frac{Y_{j,t}}{Y_t} dj \right]$$

- Zero-profit condition

$$P_t = \int_0^1 P_{i,t} G'^{-1} \left[\frac{P_{i,t}}{P_t} \int_0^1 G' \left(\frac{Y_{j,t}}{Y_t} ; \varepsilon_t^p \right) \frac{Y_{j,t}}{Y_t} dj \right] di \tag{7}$$

Intermediate Goods Producers

- F.O.C. for cost minimization

$$K_t^s = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}L_t \tag{8}$$

- Marginal cost(Lagrangian multiplier for production function)

$$MC_t = \frac{W_t}{(1-\alpha)\gamma^{(1-\alpha)t}\mathrm{e}^{\varepsilon_t^a}(K_t^s/L_t)^{\alpha}} = \frac{W_t^{1-\alpha}(R_t^k)^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\gamma^{(1-\alpha)t}\mathrm{e}^{\varepsilon_t^a}} \tag{9}$$

- F.O.C. for price setting

$$E_t\sum_{s=0}^{\infty}\xi_p^s\frac{\beta^s\Xi_{t+s}P_t}{\Xi_t P_{t+s}}Y_{i,t+s}(\eta_{t+s}^p(\cdot)-1)\left[\tilde{P}_{i,t}X_{t,s}^p - \frac{\eta_{t+s}^p(\cdot)}{\eta_{t+s}^p(\cdot)-1}MC_{t+s}\right] = 0 \tag{10}$$

where

$$\eta_{t+s}^p\left(\frac{\tilde{P}_{i,t}X_{t,s}^p}{P_{t+s}};\varepsilon_{t+s}^p\right) \equiv -\frac{1}{G'^{-1}(\cdot)}\frac{G'(\cdot)}{G''(\cdot)}$$

- since $\tilde{P}_{i,t}$ is the same, from equation (7), final goods price is

$$P_t = (1-\xi_p)\tilde{P}_t G'^{-1}\left[\frac{\tilde{P}_t}{P_t}\tau_t^p\right] + \xi_p\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}P_{t-1}G'^{-1}\left[\frac{\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}P_{t-1}}{P_t}\tau_t^p\right] \tag{11}$$

Household

- F.O.C.

$$(\partial C_t)\Xi_t = (C_t - \lambda C_{t-1})^{-\sigma_c} \mathrm{e}^{\frac{\sigma_c - 1}{1+\sigma_l}\bar{L}_t^{1+\sigma_l}} \tag{12}$$

$$(\partial \bar{L}_t)\frac{W_t^h}{P_t} = (C_t - \lambda C_{t-1})\bar{L}_t^{\sigma_l} \tag{13}$$

$$(\partial B_t)\Xi_t = \beta \mathrm{e}^{\varepsilon_t^b} R_t E_t \left[\frac{\Xi_{t+1}}{\pi_{t+1}}\right] \tag{14}$$

$$\begin{aligned}(\partial I_t)1 = Q_t \mathrm{e}^{\varepsilon_t^i}&\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] \\ &+ \beta E_t \frac{\Xi_{t+1}}{\Xi_t}\left[Q_{t+1}\mathrm{e}^{\varepsilon_{t+1}^i} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]\end{aligned} \tag{15}$$

$$(\partial K_t)Q_t = \beta E_t \frac{\Xi_{t+1}}{\Xi_t}\left[\left(\frac{R_{t+1}^k Z_{t+1}}{P_{t+1}} - a(Z_{t+1})\right) + Q_{t+1}(1-\delta)\right] \tag{16}$$

$$(\partial Z_t)\frac{R_t^k}{P_t} = a'(Z_t) \tag{17}$$

where $Q_t \equiv \frac{\Xi_t^k}{\Xi_t}$, and Ξ_t^k is Lagrangian multiplier for capital K_t.

Labor Sector

- Zero-profit condition for labor packers

$$W_t = \int_0^1 W_{l,t} H'^{-1} \left[\frac{W_{l,t}}{W_t} \int_0^1 H' \left(\frac{L_{j,t}}{L_t}; \varepsilon_t^w \right) \frac{L_{j,t}}{L_t} dj \right] dl \tag{18}$$

- F.O.C. for labor unions

$$E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} L_{l,t+s} (\eta_{t+s}^w(\cdot) - 1) \left[\tilde{W}_{l,t} X_{t,s}^w - \frac{\eta_{t+s}^w(\cdot)}{\eta_{t+s}^w(\cdot) - 1} W_{t+s}^h \right] = 0 \tag{19}$$

where

$$\eta_{t+s}^w \left(\frac{\tilde{W}_{l,t} X_{t,s}^w}{W_{t+s}}; \varepsilon_{t+s}^w \right) \equiv -\frac{1}{H'^{-1}(\cdot)} \frac{H'(\cdot)}{H''(\cdot)}$$

- since $\tilde{W}_{l,t}$ is the same, from equation (18), final wage is

$$W_t = (1 - \xi_w) \tilde{W}_t H'^{-1} \left[\frac{\tilde{W}_t}{W_t} \tau_t^w \right] + \xi_w \gamma \pi_{t-1}^{\iota_w} \pi^{1-\iota_w} W_{t-1} H'^{-1} \left[\frac{\gamma \pi_{t-1}^{\iota_w} \pi^{1-\iota_w} W_{t-1}}{W_t} \tau_t^w \right] \tag{20}$$

where

$$\tau_t^w \equiv \int_0^1 H' \left(\frac{L_{l,t}}{L_t}; \varepsilon_t^w \right) \frac{L_{l,t}}{L_t} dl$$

Resource Constraint

- Resource Constraint

$$C_t + I_t + G_t + a(Z_t)K_{t-1} = Y_t \tag{21}$$

Detrending Notations

- Detrending notations

$$
\begin{aligned}
c_t &= \frac{C_t}{\gamma^t} & \tilde{p}_t &= \frac{\tilde{P}_t}{P_t} \\
i_t &= \frac{I_t}{\gamma^t} & r_t^k &= \frac{R_t^k}{P_t} \\
y_t &= \frac{Y_t}{\gamma^t} & mc_t &= \frac{MC_t}{P_t} \\
y_{i,t} &= \frac{Y_{i,t}}{\gamma^t} & w_t &= \frac{W_t}{P_t \gamma^t} \\
y_t^p &= \frac{Y_t^p}{\gamma^t} & w_t^h &= \frac{W_t^h}{P_t \gamma^t} \\
k_t^s &= \frac{K_t^s}{\gamma^t} & \zeta_t &= \Xi_t \gamma^{\sigma_c t} \\
k_{i,t}^s &= \frac{K_{i,t}^s}{\gamma^t} \\
k_t &= \frac{K_t}{\gamma^t}
\end{aligned}
$$

Final Goods Producers

- From equation (1), we get

$$\int_0^1 G\left(\frac{y_{i,t}}{y_t};\varepsilon_t^p\right)di = 1 \tag{22}$$

- From equation (11), we get

$$1 = (1-\xi_p)\tilde{p}_t G'^{-1}(\tilde{p}_t\tau_t^p) + \xi_p \pi_{t-1}^{\iota_p}\pi^{1-\iota_p}\pi_t^{-1}G'^{-1}(\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}\pi_t^{-1}\tau_t^p) \tag{23}$$

Intermediate Goods Producers

- From equation (2), we get

$$y_{i,t} = \mathrm{e}^{\varepsilon_t^a}(k_{i,t}^s)^\alpha (L_{i,t})^{1-\alpha} - \Phi \tag{24}$$

- From equation (8), we get

$$k_t^s = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}L_t \tag{25}$$

- From equation (9), we get

$$mc_t = \frac{w_t}{(1-\alpha)\mathrm{e}^{\varepsilon_t^a}(k_t^s/L_t)^\alpha} = \frac{w_t^{1-\alpha}(r_t^k)^\alpha}{\alpha^\alpha(1-\alpha)^{1-\alpha}\mathrm{e}^{\varepsilon_t^a}} \tag{26}$$

- From equation (10) (divided by P_t), we get

$$E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\gamma^{(1-\sigma_c)s}\frac{\zeta_{t+s}}{\zeta_t}y_{i,t+s}(\eta_{t+s}^p(\cdot)-1)\left[\tilde{p}_{i,t}\frac{\prod_{l=1}^s \pi_{t+l-1}^{\iota_p}\pi^{1-\iota_p}}{\prod_{l=1}^s \pi_{t+l}} - \frac{\eta_{t+s}^p\left(\tilde{p}_{i,t}\frac{\prod_{l=1}^s \pi_{t+l-1}^{\iota_p}\pi^{1-\iota_p}}{\prod_{l=1}^s \pi_{t+l}};\varepsilon_{t+s}^p\right)}{\eta_{t+s}^p(\cdot)-1}mc_{t+s}\right] = 0 \tag{27}$$

Household

- From equation (4), we get

$$k_t = \frac{(1-\delta)}{\gamma} k_{t-1} + \mathrm{e}^{\varepsilon_t^i} \left[1 - S(\frac{i_t \gamma}{i_{t-1}}) \right] i_t \tag{28}$$

- From equation (5), we get

$$k_t^s = \frac{Z_t k_{t-1}}{\gamma} \tag{29}$$

- From equations (12) to (17),

$$\zeta_t = (c_t - \frac{\lambda}{\gamma} c_{t-1})^{-\sigma_c} \mathrm{e}^{\frac{\sigma_c - 1}{1+\sigma_l} \bar{L}_t^{1+\sigma_l}} \tag{30}$$

$$w_t^h = (c_t - \frac{\lambda}{\gamma} c_{t-1}) \bar{L}_t^{\sigma_l} \tag{31}$$

$$\zeta_t = \beta \gamma^{-\sigma_c} \mathrm{e}^{\varepsilon_t^b} R_t E_t \left[\frac{\zeta_{t+1}}{\pi_{t+1}} \right] \tag{32}$$

$$1 = Q_t \mathrm{e}^{\varepsilon_t^i} \left[1 - S\left(\frac{i_t \gamma}{i_{t-1}}\right) - S'\left(\frac{i_t \gamma}{i_{t-1}}\right) \frac{i_t \gamma}{i_{t-1}} \right] + \beta \gamma^{-\sigma_c} E_t \frac{\zeta_{t+1}}{\zeta_t} \left[Q_{t+1} \mathrm{e}^{\varepsilon_{t+1}^i} S'\left(\frac{i_{t+1}\gamma}{i_t}\right) \left(\frac{i_{t+1}\gamma}{i_t}\right)^2 \right] \tag{33}$$

$$Q_t = \beta \gamma^{-\sigma_c} E_t \frac{\zeta_{t+1}}{\zeta_t} \left[r_{t+1}^k Z_{t+1} - a(Z_{t+1}) + Q_{t+1}(1-\delta) \right] \tag{34}$$

$$r_t^k = a'(Z_t) \tag{35}$$

Labor Sector

- From equation (19), we get

$$E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \gamma^{(1-\sigma_c)s} \frac{\zeta_{t+s}}{\zeta_t} L_{l,t+s}(\eta_{t+s}^w(\cdot) - 1) \left[\tilde{w}_{l,t} \frac{\prod_{l=1}^s \pi_{t+l-1}^{\iota_w} \pi^{1-\iota_w}}{\prod_{l=1}^s \pi_{t+l}} - \frac{\eta_{t+s}^w \left(\frac{\tilde{w}_{l,t}}{w_{t+s}} \frac{\prod_{l=1}^s \pi_{t+l-1}^{\iota_w} \pi^{1-\iota_w}}{\prod_{l=1}^s \pi_{t+l}} ; \varepsilon_{t+s}^p \right)}{\eta_{t+s}^w(\cdot) - 1} w_{t+s}^h \right] = 0 \quad (36)$$

- From equation (20), we get

$$w_t = (1-\xi_w) \tilde{w}_t H'^{-1} \left[\frac{\tilde{w}_t}{w_t} \tau_t^w \right] + \xi_w \pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \pi_t^{-1} w_{t-1} H'^{-1} \left[\frac{\pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \pi_t^{-1} w_{t-1}}{w_t} \tau_t^w \right] \quad (37)$$

Government

- From equation (6), we get

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho} \left[\left(\frac{\pi_t}{\pi}\right)^{r_\pi} \left(\frac{y_t}{y_t^p}\right)^{r_y}\right]^{1-\rho} \left(\frac{y_t/y_{t-1}}{y_t^p/y_{t-1}^p}\right)^{r_{\Delta y}} \mathrm{e}^{\varepsilon_t^r} \qquad (38)$$

Resource Constraint

- From equation (21), we get

$$c_t + i_t + \mathrm{e}^{\varepsilon_t^g} g_y y + \frac{a(Z_t)}{\gamma} k_{t-1} = y_t \tag{39}$$

Steady State

- Steady state relationship

$$
\begin{aligned}
&\tilde{p} = 1 \\
&z = 1 \\
&a(1) = 0 \\
&q = 1 \\
&S(\gamma) = S'(\gamma) = 0 \\
&S''(\gamma) = \varphi \\
&\tfrac{a'(1)}{a''(1)} = \tfrac{1-\psi}{\psi} \\
&w = \tilde{w} \\
&y = y_i = (k^s)^\alpha L^{1-\alpha} - \Phi \\
&L = \bar{L} = L_l
\end{aligned}
$$

Steady State

- Steady state values for log-linearization $(r, r^k, c/y, i/y, k/y, wL/c)$
- r. From equation (32), we get

$$r = \frac{\pi}{\beta\gamma^{-\sigma_c}}$$

- r^k. From equation (34), we get

$$r^k = \frac{1}{\beta\gamma^{-\sigma_c}} - (1-\delta)$$

Steady State

- k/y. From zero-profit conditions for intermediate goods producers $(wL_i + r^k k_i^s = mc(y_i + \Phi) = \tilde{p} y_i)$, we get

$$\phi_p \equiv \frac{\Phi + y}{y} = \frac{1}{mc} = \frac{\eta^p}{\eta^p - 1} = \frac{(k^s)^\alpha L^{1-\alpha}}{y}$$

From equation (26), we get

$$w = \left[\frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{\phi_p (r^k)^\alpha} \right]^{\frac{1}{1-\alpha}}$$

From equation (25), we get

$$\frac{k^s}{L} = \frac{\alpha}{1-\alpha} \frac{w}{r^k}$$

From equation (24), we get

$$\frac{k^s}{y} = \phi_p \left(\frac{k^s}{L} \right)^{1-\alpha}$$

From equation (29), we get

$$\frac{k}{y} = \gamma \frac{k^s}{y}$$

Steady State

- i/y. From equation (28), we get

$$\frac{i}{y} = \frac{i}{k}\frac{k}{y} = (\gamma - 1 + \delta)\frac{k^s}{y}$$

- c/y. From equation (39), we get

$$\frac{c}{y} = 1 - \frac{i}{y} - g_y$$

- wL/c. From equation (36), we get

$$w^h = \frac{\eta_w - 1}{\eta_w} w = \frac{w}{\phi_w}$$

Then from equation (25)

$$\frac{w^h L}{c} = \frac{w}{\phi_w}\left(\frac{L}{k^s}\right)\left(\frac{k^s}{y}\right)\left(\frac{y}{c}\right) = \frac{1}{\phi_w}\frac{1-\alpha}{\alpha}\frac{r^k (k^s/y)}{c/y}$$

Log-Linearization

- SW(1). From equation (39), we get

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + \frac{r^k}{\gamma}\frac{k}{y}\hat{z}_t + \varepsilon_t^g$$

- SW(2). From equations (30) and (32),

$$\hat{c}_t = \frac{\frac{\lambda}{\gamma}}{1+\frac{\lambda}{\gamma}}\hat{c}_{t-1} + \frac{1}{1+\frac{\lambda}{\gamma}}E\hat{c}_{t+1} + \frac{\sigma_c - 1}{\sigma_c\left(1+\frac{\lambda}{\gamma}\right)}\frac{w^h L}{c}(\hat{l}_t - E\hat{l}_{t+1})$$
$$- \frac{1-\frac{\lambda}{\gamma}}{\sigma_c\left(1+\frac{\lambda}{\gamma}\right)}(\hat{r}_t - E\hat{\pi}_{t+1} + \varepsilon_t^b)$$

- SW(3). From equation (33), we get

$$\hat{i}_t = \frac{1}{1+\beta\gamma^{1-\sigma_c}}\hat{i}_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}}E\hat{i}_{t+1} + \frac{1}{(1+\beta\gamma^{1-\sigma_c})\gamma^2\varphi}\hat{q}_t + \varepsilon_t^i$$

Log-Linearization

- SW(4). From equations (34) and (32), we get

$$\hat{q}_t = \beta\gamma^{-\sigma_c}(1-\delta)E\hat{q}_{t+1} + \beta\gamma^{-\sigma_c}r^k E\hat{r}^k_{t+1} - (\hat{r}_t - E\hat{\pi}_{t+1} + \varepsilon^b_t)$$

- SW(5). From equations (22) and (24),

$$\hat{y}_t = \phi_p(\alpha\hat{k}^s_t + (1-\alpha)\hat{l}_t + \varepsilon^a_t)$$

- SW(6). From equation (29), we get

$$\hat{k}^s_t = \hat{k}_{t-1} + \hat{z}_t$$

- SW(7). From equation (35), we get

$$\hat{z}_t = \frac{a'(1)}{a''(1)}\hat{r}^k_t = \frac{1-\psi}{\psi}\hat{r}^k_t$$

- SW(8). From equation (28), we get

$$\hat{k}_t = \frac{1-\delta}{\gamma}\hat{k}_{t-1} + \left(1 - \frac{1-\delta}{\gamma}\right)\hat{i}_t + \left(1 - \frac{1-\delta}{\gamma}\right)(1+\beta\gamma^{1-\sigma_c})\gamma^2\varphi\varepsilon^i_t$$

Log-Linearization

- SW(9). From equation (26), we get price mark-up

$$\mu_t^p \equiv -\hat{mc}_t = \alpha(\hat{k}_t^s - \hat{l}_t) + \varepsilon_t^a - \hat{w}_t$$

- SW(10). From equations (23) and (27), we get

$$\hat{\pi}_t = \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p}\hat{\pi}_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}E\hat{\pi}_{t+1} - \frac{1-\xi_p\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}\frac{1-\xi_p}{\xi_p}\frac{1}{1+(\phi_p-1)\varepsilon_p}\mu_t^p + \varepsilon_t^p$$

where $\varepsilon_p \equiv \frac{\eta^{p\prime}}{\eta^p}$.

- SW(11). From equation (25), we get

$$\hat{r}_t^k = -(\hat{k}_t^s - \hat{l}_t) + \hat{w}_t$$

Log-Linearization

- SW(12). From equation (31),

$$\mu_t^w \equiv \hat{w}_t - \hat{w}_t^h = \hat{w}_t - \left(\sigma_l \hat{l}_t + \frac{1}{1-\frac{\lambda}{\gamma}}(\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1})\right)$$

- SW(13). From equations (37) and (36),

$$\begin{aligned}\hat{w}_t = \frac{1}{1+\beta\gamma^{1-\sigma_c}}\hat{w}_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}}(E\hat{w}_{t+1} + E\hat{\pi}_{t+1}) \\ -\frac{1+\beta\gamma^{1-\sigma_c}\iota_w}{1+\beta\gamma^{1-\sigma_c}}\hat{\pi}_t + \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}}\hat{\pi}_{t-1} \\ -\frac{1-\xi_w\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}}\frac{1-\xi_w}{\xi_w}\frac{1}{1+(\phi_w-1)\varepsilon_w}\mu_t^w + \varepsilon_t^w\end{aligned}$$

  where $\phi_w \equiv \frac{\eta^w}{\eta^w-1}$ and $\varepsilon_w \equiv \frac{\eta^{w\prime}}{\eta^w}$.
- SW(14). From equation (38), we get

$$\hat{r}_t = \rho\hat{r}_{t-1} + (1-\rho)[r_\pi\hat{\pi}_t + r_y(\hat{y}_t - \hat{y}_t^p)] + r_{\Delta y}[(\hat{y}_t - \hat{y}_t^p) - (\hat{y}_{t-1} - \hat{y}_{t-1}^p)] + \varepsilon_t^r$$

Log-Linearization

- 14 equations: SW(1) - SW(14)
- 14 variables: $\hat{y}_t$, $\hat{c}_t$, $\hat{i}_t$, $\hat{q}_t$, $\hat{k}_t^s$, $\hat{k}_t$, $\hat{z}_t$, $\hat{r}_t^k$, $\hat{\mu}_t^p$, $\hat{\pi}_t$, $\hat{\mu}_t^w$, $\hat{w}_t$, $\hat{l}_t$, $\hat{r}_t$.

Outline

Observable Variables and Exogenous Shocks

Observable Variables(7)	**Exogenous Shocks(7)**
real GDP(log diff)	exogenous spending shock(ε_t^g)
real consumption(log diff)	risk premium shock(ε_t^b)
real investment(log diff)	investment-specific technology shock(ε_t^i)
real wage(log diff)	wage mark-up shock(ε_t^w)
hours worked(log)	total factor productivity shock(ε_t^a)
GDP deflator(log diff)	price mark-up shock(ε_t^p)
federal fund rate	monetary policy shock(ε_t^r)

Measurement Equation

$$Y_t = \begin{bmatrix} dlGDP_t \\ dlCON_t \\ dlINV_t \\ dlWAG_t \\ lHOURS_t \\ dlP_t \\ FEDFUNDS_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{l}_t \\ \hat{\pi}_t \\ \hat{r}_t \end{bmatrix}$$

where l and dl stand for 100 times log and log difference, respectively; $\bar{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate to real GDP, consumption, investment and wages; $\bar{\pi} = 100(\pi - 1)$ is the quarterly steady-state inflation rate; and $\bar{r} = (\beta^{-1}\gamma^{\sigma_c}\pi - 1)$ is the steady-state nominal interest rate; $\bar{l}$ is steady-state hours worked, which is normalized to be equal to zero.

Del Negro and Schorfheide (2008 JME)

Three groups of parameters

- The first group: steady states (ratios, or other long-run measures)
- The second group: endogenous propagation mechanism (taste, technology, and policy parameters)
- The third group: propagation mechanism of the exogenous shocks (autocorrelations, standard deviations)

Estimation Strategy

- Fixed parameters (5)
  - Depreciation rate $\delta = 0.025$
  - Exogenous spending-GDP ratio $g_y = 0.18$
  - Steady-state mark-up in the labor market $\phi_w = 1.5$
  - Curvature parameter of Kimball aggregator in the goods market $\varepsilon_p = 10$
  - Curvature parameter of Kimball aggregator in the labor market $\varepsilon_w = 10$

Priors and Posteriors (1/2)

TABLE 1A—PRIOR AND POSTERIOR DISTRIBUTION OF STRUCTURAL PARAMETERS

	Prior distribution			Posterior distribution			
	Distr.	Mean	St. Dev.	Mode	Mean	5 percent	95 percent
φ	Normal	4.00	1.50	5.48	5.74	3.97	7.42
σ_c	Normal	1.50	0.37	1.39	1.38	1.16	1.59
h	Beta	0.70	0.10	0.71	0.71	0.64	0.78
ξ_w	Beta	0.50	0.10	0.73	0.70	0.60	0.81
σ_l	Normal	2.00	0.75	1.92	1.83	0.91	2.78
ξ_p	Beta	0.50	0.10	0.65	0.66	0.56	0.74
ι_w	Beta	0.50	0.15	0.59	0.58	0.38	0.78
ι_p	Beta	0.50	0.15	0.22	0.24	0.10	0.38
ψ	Beta	0.50	0.15	0.54	0.54	0.36	0.72
Φ	Normal	1.25	0.12	1.61	1.60	1.48	1.73
r_π	Normal	1.50	0.25	2.03	2.04	1.74	2.33
ρ	Beta	0.75	0.10	0.81	0.81	0.77	0.85
r_y	Normal	0.12	0.05	0.08	0.08	0.05	0.12
$r_{\Delta y}$	Normal	0.12	0.05	0.22	0.22	0.18	0.27
$\bar{\pi}$	Gamma	0.62	0.10	0.81	0.78	0.61	0.96
$100(\beta^{-1} - 1)$	Gamma	0.25	0.10	0.16	0.16	0.07	0.26
$\bar{l}$	Normal	0.00	2.00	−0.1	0.53	−1.3	2.32
$\bar{\gamma}$	Normal	0.40	0.10	0.43	0.43	0.40	0.45
α	Normal	0.30	0.05	0.19	0.19	0.16	0.21

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Priors and Posteriors (2/2)

TABLE 1B—PRIOR AND POSTERIOR DISTRIBUTION OF SHOCK PROCESSES

	Prior distribution			Posterior distribution			
	Distr.	Mean	St. Dev.	Mode	Mean	95 percent	5 percent
σ_a	Invgamma	0.10	2.00	0.45	0.45	0.41	0.50
σ_b	Invgamma	0.10	2.00	0.24	0.23	0.19	0.27
σ_g	Invgamma	0.10	2.00	0.52	0.53	0.48	0.58
σ_I	Invgamma	0.10	2.00	0.45	0.45	0.37	0.53
σ_r	Invgamma	0.10	2.00	0.24	0.24	0.22	0.27
σ_p	Invgamma	0.10	2.00	0.14	0.14	0.11	0.16
σ_w	Invgamma	0.10	2.00	0.24	0.24	0.20	0.28
ρ_a	Beta	0.50	0.20	0.95	0.95	0.94	0.97
ρ_b	Beta	0.50	0.20	0.18	0.22	0.07	0.36
ρ_g	Beta	0.50	0.20	0.97	0.97	0.96	0.99
ρ_I	Beta	0.50	0.20	0.71	0.71	0.61	0.80
ρ_r	Beta	0.50	0.20	0.12	0.15	0.04	0.24
ρ_p	Beta	0.50	0.20	0.90	0.89	0.80	0.96
ρ_w	Beta	0.50	0.20	0.97	0.96	0.94	0.99
μ_p	Beta	0.50	0.20	0.74	0.69	0.54	0.85
μ_w	Beta	0.50	0.20	0.88	0.84	0.75	0.93
ρ_{ga}	Beta	0.50	0.20	0.52	0.52	0.37	0.66

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Outline

1. Introduction
2. Model
3. Estimation
4. Post-Estimation Analyses
5. Conclusion

Forecast Performance (1/2)

TABLE 2—COMPARISON OF THE MARGINAL LIKELIHOOD OF ALTERNATIVE VAR MODELS AND THE DSGE MODEL

Order of the VAR	No other prior	Sims and Zha (1998) prior
VAR(1)	−928.0	−940.9
VAR(2)	−966.6	−915.8
VAR(3)	−1018.1	−908.7
VAR(4)	−1131.2	−906.6
VAR(5)	–	−907.7
Memo: DSGE model	−905.8	−905.8

Note: In order to increases the comparability of the marginal likelihood of the various models, all models are estimated using the period 1956:1–1965:4 as a training sample (Sims 2003).

Forecast Performance (2/2)

TABLE 3—OUT-OF-SAMPLE PREDICTION PERFORMANCE

	GDP	dP	Fedfunds	Hours	Wage	CONS	INV	Overall
VAR(1)	*RMSE-statistic for different forecast horizons*							
1q	0.60	0.25	0.10	0.46	0.64	0.60	1.62	−12.87
2q	0.94	0.27	0.18	0.78	1.02	0.95	2.96	−8.19
4q	1.64	0.34	0.36	1.45	1.67	1.54	5.67	−3.25
8q	2.40	0.53	0.64	2.13	2.88	2.27	8.91	1.47
12q	2.78	0.63	0.79	2.41	4.09	2.74	10.97	2.36
BVAR(4)	*Percentage gains (+) or losses (−) relative to VAR*(1) *model*							
1q	2.05	14.14	−1.37	−3.43	2.69	12.12	2.54	3.25
2q	−2.12	15.15	−16.38	−7.32	−0.29	10.07	2.42	0.17
4q	−7.21	31.42	−12.61	−8.58	−3.82	1.42	0.43	0.51
8q	−15.82	33.36	−13.26	−13.94	−8.98	−8.19	−11.58	−4.10
12q	−15.55	37.59	−13.56	−4.66	−15.87	−3.10	−23.49	−9.84
DSG	*Percentage gains (+) or losses (−) relative to VAR*(1) *model*							
1q	5.68	2.05	−8.24	0.68	5.99	20.16	9.22	3.06
2q	14.93	10.62	−17.22	10.34	6.20	25.85	16.79	2.82
4q	20.17	46.21	1.59	19.52	9.21	26.18	21.42	6.82
8q	22.55	68.15	28.33	22.34	15.72	21.82	25.95	11.50
12q	32.17	74.15	40.32	27.05	21.88	23.28	41.61	13.51

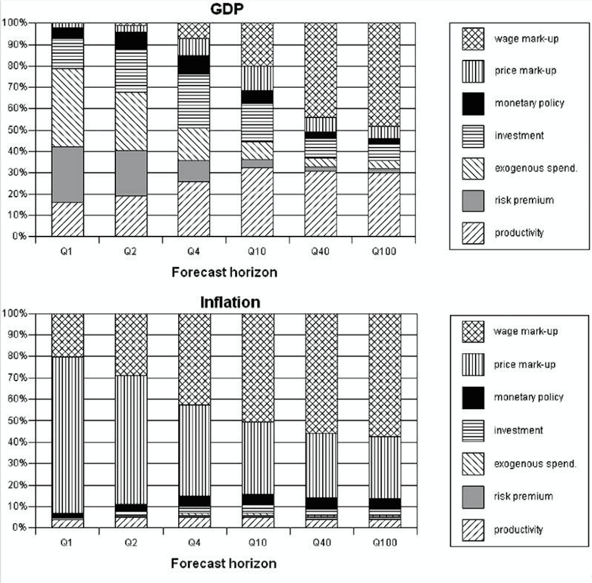
Notes: All models are estimated starting in 1966:1. The forecast period is 1990:1–2004:4. VAR(1) and BVAR(4) models are reestimated each quarter, the DSGE model each year. The overall measure of forecast performance is the log determinant of the uncentered forecast error covariance matrix. Gains and losses in the overall measure are expressed as the difference in the overall measure divided by the number of variables and by two to convert the variance to standard errors (times 100).

Model Sensitivity

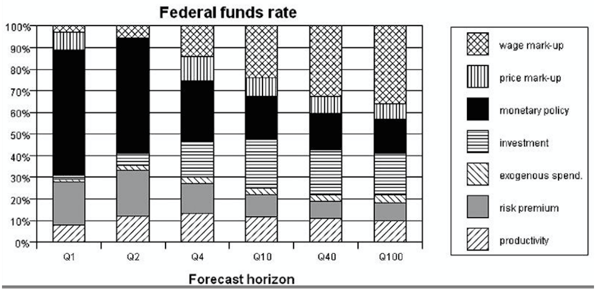
TABLE 4—TESTING THE EMPIRICAL IMPORTANCE OF THE NOMINAL AND REAL FRICTIONS IN THE DSGE MODEL

	Base	$\xi_p = 0.1$	$\xi_w = 0.1$	$\iota_p = 0.0$	$\iota_w = 0.0$	$\varphi = 0.1$	$h = 0.1$	$\psi = 0.99$	$\Phi = 1.1$
Marginal likelihood									
	−923	−975	−973	−918	−927	−1084	−959	−924	−949
Mode of the structural parameters									
φ	5.48	4.41	2.78	5.45	5.62	0.10	1.26	5.33	5.19
σ_c	1.39	1.31	1.80	1.43	1.42	2.78	1.90	1.39	1.27
h	0.71	0.70	0.34	0.70	0.71	0.12	0.10	0.70	0.71
ξ_w	0.73	0.55	0.10	0.75	0.75	0.89	0.73	0.73	0.78
σ_l	1.92	1.48	0.25	1.91	1.91	5.24	1.21	1.79	2.33
ξ_p	0.65	0.10	0.48	0.66	0.69	0.86	0.62	0.59	0.80
ι_w	0.59	0.71	0.68	0.61	0.01	0.39	0.61	0.63	0.58
ι_p	0.22	0.84	0.24	0.01	0.24	0.08	0.21	0.21	0.19
ψ	0.54	0.82	0.66	0.54	0.50	0.02	0.69	0.99	0.45
Φ	1.61	1.79	1.64	1.60	1.61	1.15	1.44	1.62	1.10
r_π	2.03	2.15	2.15	2.01	2.01	2.03	2.24	2.04	1.98
ρ	0.81	0.79	0.75	0.81	0.82	0.84	0.81	0.80	0.80
r_y	0.08	0.08	0.08	0.08	0.09	0.23	0.12	0.08	0.10
$r_{\Delta y}$	0.22	0.21	0.25	0.22	0.22	0.30	0.29	0.23	0.25
α	0.19	0.21	0.20	0.19	0.19	0.20	0.19	0.18	0.13
Mode of the autoregressive parameters of the exogenous shock processes									
ρ_a	0.95	0.96	0.97	0.96	0.95	0.99	0.97	0.96	0.96
ρ_b	0.18	0.19	0.67	0.18	0.18	0.89	0.79	0.18	0.28
ρ_g	0.97	0.96	0.97	0.97	0.97	0.99	0.97	0.97	0.96
ρ_I	0.71	0.71	0.78	0.70	0.69	0.99	0.90	0.73	0.74
ρ_r	0.12	0.14	0.13	0.12	0.11	0.02	0.03	0.13	0.11
ρ_p	0.90	0.97	0.94	0.88	0.88	0.60	0.93	0.92	0.85
ρ_w	0.97	0.98	0.98	0.97	0.97	0.92	0.98	0.97	0.95
μ_p	0.74	0.20	0.71	0.59	0.77	0.34	0.76	0.71	0.67
μ_w	0.88	0.75	0.14	0.91	0.88	0.96	0.95	0.90	0.87

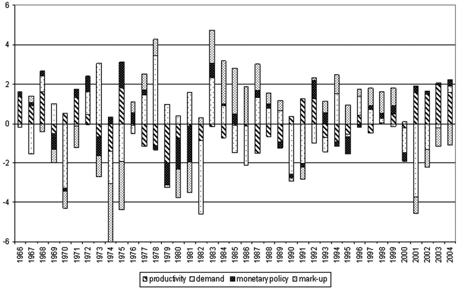
Variance Decomposition (1/2)

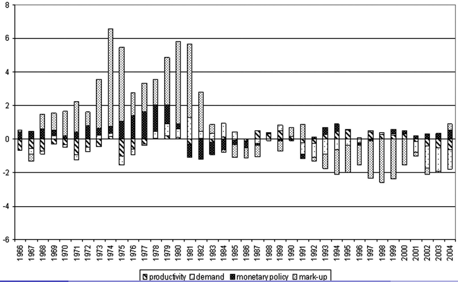


Variance Decomposition (2/2)

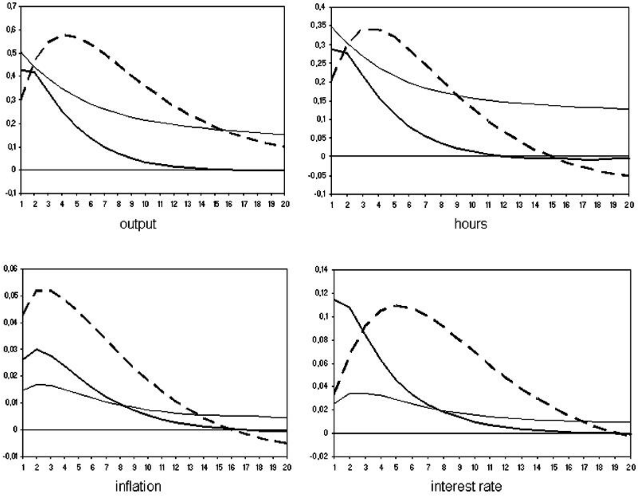


Historical Decomposition: GDP and Inflation



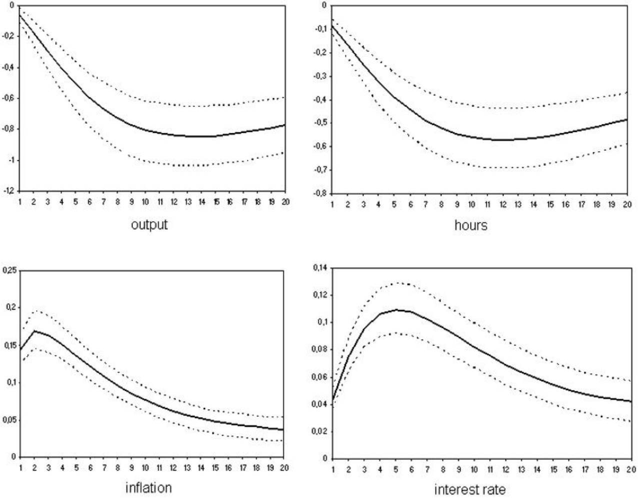


IRF: Demand Shocks



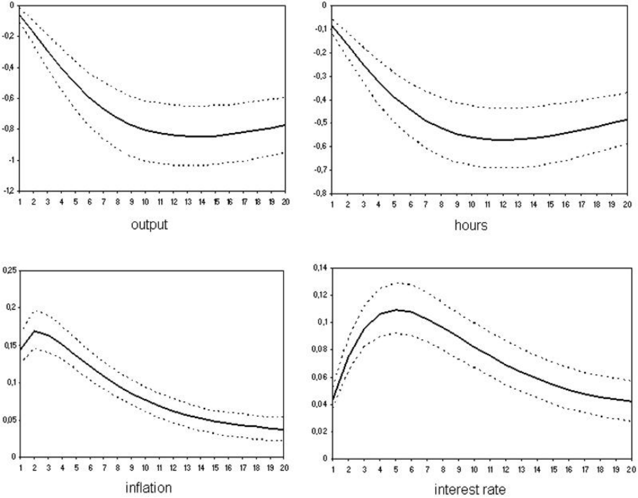
Note: Bold solid line: risk premium shock; thin solid line: exogenous spending shock; dashed line: investment shock.

IRF: Wage Mark-up Shock



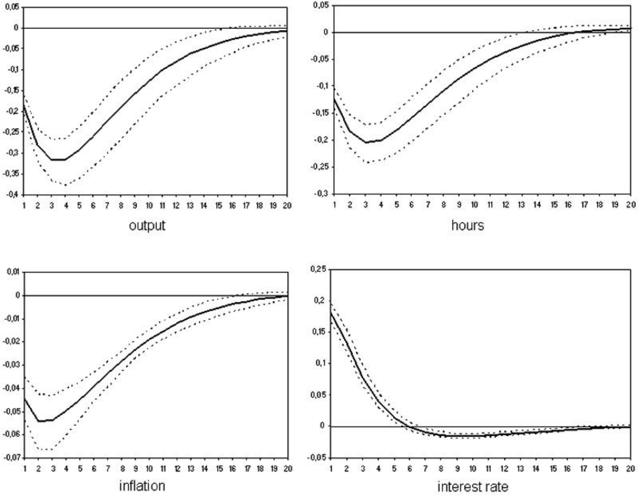
Note: The solid line is the mean impulse response; the dotted lines are the 10 percent and 90 percent posterior intervals.

IRF: Wage Mark-up Shock



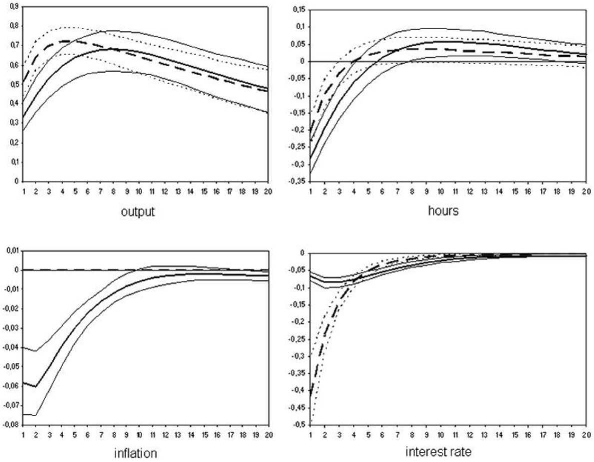
Note: The solid line is the mean impulse response; the dotted lines are the 10 percent and 90 percent posterior intervals.

IRF: Monetary Policy Shock



Note: The solid line is the mean impulse response; the dotted lines are the 10 percent and 90 percent posterior intervals.

IRF: TFP Shock



Note: The solid lines represent the estimated actual mean responses and the 10 percent and 90 percent posterior interval; the dashed lines represent the counterfactual flexible- wage-and-price responses.

Subsample Estimates: the "Great Inflation" vs the "Great Moderation"

TABLE 5—SUBSAMPLE ESTIMATES

	Structural parameters					Shock processes			
	1966:1–1979:2		1984:1–2004:4			1966:1–1979:2		1984:1–2004:4	
	Mode	SD	Mode	SD		Mode	SD	Mode	SD
φ	3.61	1.03	6.23	1.12	σ_a	0.58	0.05	0.35	0.02
σ_c	1.39	0.22	1.47	0.13	σ_b	0.22	0.04	0.18	0.02
h	0.63	0.07	0.68	0.04	σ_g	0.54	0.05	0.41	0.03
ξ_w	0.65	0.07	0.74	0.13	σ_I	0.52	0.09	0.39	0.05
σ_l	1.52	0.65	2.30	0.67	σ_r	0.20	0.02	0.12	0.01
ξ_p	0.55	0.08	0.73	0.04	σ_p	0.22	0.03	0.11	0.01
ι_w	0.58	0.13	0.46	0.16	σ_w	0.20	0.02	0.21	0.03
ι_p	0.45	0.18	0.21	0.09	ρ_a	0.97	0.01	0.94	0.02
ψ	0.34	0.13	0.69	0.11	ρ_b	0.39	0.17	0.14	0.08
Φ	1.43	0.09	1.54	0.09	ρ_g	0.91	0.03	0.96	0.01
r_π	1.65	0.19	1.77	0.29	ρ_I	0.60	0.10	0.64	0.07
ρ	0.81	0.03	0.84	0.02	ρ_r	0.22	0.10	0.29	0.10
r_y	0.17	0.03	0.08	0.05	ρ_p	0.51	0.24	0.74	0.13
$r_{\Delta y}$	0.20	0.03	0.16	0.02	ρ_w	0.96	0.02	0.82	0.15
$\bar{\pi}$	0.72	0.11	0.67	0.10	μ_p	0.46	0.20	0.59	0.18
$\beta^{-1}-1$	0.14	0.06	0.12	0.05	μ_w	0.84	0.07	0.62	0.17
$\bar{l}$	0.03	0.62	−0.55	1.21	ρ_{ga}	0.58	0.11	0.39	0.11
$\bar{\gamma}$	0.33	0.04	0.44	0.02					
α	0.19	0.02	0.21	0.02					

Note: SD stands for standard deviation.

Outline

Main Take-aways (1/2)

- Apply New Keynesian model to explain main features of the US macro data: real GDP, hours worked, consumption, investment, real wages, prices, and the short-term nominal interest rate.
- Analyze the roles of frictions
  - Price and wage stickiness are found to be equally important.
  - Indexation is relatively unimportant in both goods and labor markets.
  - The most important are the investment adjustment costs.

Main Take-aways (2/2)

- Analyze the roles of shocks
  - While "demand" shocks such as the risk premium, exogenous spending, and investment-specific technology shocks explain a significant fraction of the short-run forecast variance in output, both wage mark-up (or labor supply) and, to a lesser extent, productivity shocks explain most of its variation in the medium to long run.
  - Productivity shocks have a significant short-run negative impact on hours worked.
  - Inflation developments are mostly driven by the price mark-up shocks in the short run and the wage mark-up shocks in the long run.
  - The "Great Inflation" vs the "Great Moderation": most of the structural parameters are stable over those two periods. The biggest difference concerns the variances of the structural shocks.